tertiary ray as observed by Clark, Duane and Stifler in their experiments.[4] Since the presence of the tertiary ray from sulfur appears to be confirmed by experiments reported more recently,[2] the writer in particular performed ten experiments with sulfur as the secondary radiator (the curve for sulfur in figure 1 represents one of the results). None of these experiments showed the existence of this tertiary peak.

The writer is indebted to Prof. A. H. Compton for his interest in this work.

[1] Compton and Woo, these PROCEEDINGS, **10,** 370 (1924).

[2] Allison, Clark and Duane, these PROCEEDINGS, **10,** 370 (1914).

[3] A. H. Compton, *Physic. Rev.*, **21,** 483 (1923).

[4] Clark, Duane and Stifler, these PROCEEDINGS, **10,** 148 (1924).

THE AGE OF THE STARS

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It is well-known that the supply of energy made available by the gravitational contraction of the material of a star is insufficient, in view of the Stefan law of total radiation, to allow the time-life of a star to be more than a small fraction of the age demanded by geological considerations. Discussions of the problem of maintenance of stellar energy have been given by Shapley[1] and Russell.[2] The former discusses an hypothesis of asymmetrical radiation flow in which the rate of radiation to empty space is much less than the radiation toward other matter. Shapley also considers the destruction of mass as a possible source of stellar energy. He concludes that "... it now appears that the disagreement between the long and short time scales must be decided in favor of an exceedingly prolonged history for sidereal systems, permitting a relatively slow evolutionary development for stars and planets." Russell's paper is a very general sketch of the main outlines of the problem and is concerned mainly with the conditions under which a nuclear "unknown" source of energy comes into action.

In this paper, it is shown that the relativistic relation between energy and mass leads directly to a means of estimating the age of the stars, the method being independent of the atomic processes whereby mass as "matter" is changed into mass as "radiant energy." This relation,

$$\Delta E = c^2 \Delta m \qquad (1)$$

in which E is the energy of the system in ergs, c the fundamental constant of space-time in centimeters per second, and m is mass in grams, has been given an interpretation that is too narrow, at least in discussions of its possible applications to stellar problems. It is necessary to bear in mind that, from the manner of its derivation, the equation necessitates that loss of mass by a system accompany loss of energy as a result of radiation. We do not need to inquire into the source of the energy lost by radiation.

The point is brought out clearly in the original work of Einstein.[3] He says, "Gibt ein Körper die Energie L in Form der Strahlung ab, so verkleinert sich seine Masse um L/V^2." And again, "Die Masse eines Körpers ist ein Mass für dessen Energieinhalt; ändert sich die Energie um L, so ändert sich die Masse in demselben Sinne um $L/(9 \times 10^{20})$ wenn die Energie in Erg und die Masse in Grammen gemessen wird."

Let us assume that the amount of mass which is gained by a star during its life by means of the actual accumulation of meteoric matter is negligible and further, that there is no expulsion to infinity of matter by the star during its life. If a star is born by the accumulation of dispersed matter, the first of these assumptions is probably inadmissible in the early stages of its life, but will be true with increasing accuracy as the star gets well along in years. The second assumption is probably true at all times as the explosive disruption of a star is considered to be an exceptional occurrence.

If we grant these assumptions, at least after the period of infancy, it is evident that since there is a continual loss of energy by radiation, the mass of the star will decrease throughout its life.

The rate of radiation of energy may be taken to be in accordance with Stefan's law. This allows the equation,

$$dE/dt = 4\pi r^2 \sigma T^4 \tag{2}$$

in which dE/dt is the rate of loss of energy, r the radius of the star, σ the Stefan constant, T the effective temperature, all in C.G.S. units. Equation (1) in the derivative form is

$$dE/dt = c^2(dm/dt). \tag{3}$$

Combining (2) and (3) we may write,

$$dt = \frac{c^2 \cdot dm}{4\pi\sigma r^2 T^4} \tag{4}$$

which is the fundamental equation of this paper. It expresses the lapse of time corresponding to a differential decrease of mass in terms of known quantities.

For the computation it was convenient to use other units. Let τ be time in years, M be mass with the Sun's mass as unit, and R be the star's radius with the Sun's radius as unit. Then we have

$$\Delta t = 3.15 \times 10^7 \ \Delta\tau, \qquad m = 1.99 \times 10^{33} \ M, \qquad r = 6.95 \times 10^{10} \ R.$$

Also we have

$$\sigma = 5.72 \times 10^{-5}, \qquad c = 3.00 \times 10^{10}.$$

Using these figures, (4) gives the relation,

$$d\tau = 1.63 \times 10^{28} \cdot dM/R^2T^4 \tag{5}$$

The division of stars of given spectral type into two classes according to absolute magnitude (giants and dwarfs) presented by Russell[4] is the basis of modern theories of stellar evolution. It is usually inferred that the course of evolution of individuals is along the lines of maximum frequency on the diagram of spectral type against absolute magnitude; the individual makes his debut as a giant of M-type and roughly zero absolute magnitude proceeds to B-type at approximately constant absolute magnitude, then declines from B to M while losing in luminosity so that at M-type its absolute magnitude is about 9 or 10. But there is considerable question whether each individual actually follows this course.

Eddington[5] has shown on theoretical grounds that the maximum temperature attained by a star will depend on the mass of the star and that only the more massive stars will attain B-type. Those of lesser mass will start to decline after having reached only A or F type.

If for some reason the stars all start with approximately the same mass and this mass is sufficient to carry them to B-type, the path of individuals on the Russell diagram will be simply that of the maximum frequency on the diagram. But if there is dispersion in the initial values of the mass then some individuals will short-circuit the main path, appearing on the dwarf branch without having attained B-type. Such a short-circuiting accounts for the presence of a considerable number of individuals between the giant-line and the dwarf-line of the diagram.

In the very comprehensive discussion of the present state of knowledge of masses of the stars by Seares,[6] it is brought out that the observational data on the masses of the giants are meager. The mean masses of stars on the dwarf-line, however, seem well-defined. We may ignore, in the absence of an exact method of allowing for it, the effect of short-circuiting by small stars. This short-circuiting will affect the data in such a way as to make the decrease of mass of an individual in going from B to M type appear greater than it really is and so to make the time scale too long. Ignoring this effect, Seares' data allow the evaluation of a rough time scale along the dwarf branch.

From table XII of Seares' paper were taken the values for the effective temperature of the dwarfs of each spectral type, assuming this to be the same as for the giants in the range B_0 *to* F_0 inclusive. From Seares' table XIV were obtained the values of R ($=D$) for each type and from his table XXIII were obtained the mean values for the mass of each type.

From these data was computed the value of the integrand in (6) for each spectral type. From the values so obtained was formed a column giving the arithmetic mean of the values of the integrand for each adjacent pair of types. These means were multiplied by the corresponding ΔM to effect an approximate integration by the trapezoidal rule. Taking the origin of time as the instant when the star is of type G_0, to correspond with the choice of the Sun as unit, the integrated time lapses were computed by summing the separate time intervals.

The results of the computation follow:

TYPE	Y	M	ΔM	$\dot{y}\Delta$M	$\Sigma\bar{y}\Delta$M	TIME (YRS.)
B_0	1.78	10.	1.7	7.4	−564.	-9.2×10^{12}
B_5	6.92	8.3	2.3	28.1	557.	9.06
A_0	17.5	6.0	2.0	57.8	529.	8.62
A_5	40.1	4.0	1.5	108.	471.	7.67
F_0	104.	2.5	1.0	165.	363.	5.92
F_5	226.	1.5	0.5	198.	−198.	−3.23
G_0	568.	1.0	0.24	210	0.	0.
G_5	1180.	0.76	0.08	136.	+210.	+3.42
K_0	2220.	0.68	0.06	217.	346.	5.64
K_5	5030.	0.62	0.03	492.	563.	9.18
Ma	27900.	0.59			+1055.	+25.2

In the table, y is equal to $10^{18}/R^2T^4$. The time in years in the last column is obtained by multiplying the quantities $\Sigma\bar{y}\Delta M$ by 10^{-18} and also by 1.63×10^{28}, the factor of equation (5).

In figure 1, M is plotted as a function of the time on the basis of the third and seventh columns of the table.

There is a correspondence between the results here and some given by Shapley and Miss Cannon[7] that is worth mentioning even though it leads to considerations which cannot be developed further at present. One might suppose, at first sight, that the number of stars in a given volume of space of each spectral type would be proportional to the amount of time that each individual spends in the several spectral classes. This is the case, at least roughly, for Shapley and Miss Cannon,[7] from a study of the recently-completed Draper catalog, find the following figures for the number of dwarfs of the several types in a million cubic parsecs of the space around the solar system:

$$B, 4.4; \;\; A, 250; \;\; F, 680; \;\; \text{and}\; G, 7600.$$

Although further figures are not given, they remark, "Dwarf stars of classes K and M are probably much more numerous per unit volume than dwarf stars of class G... The most important deduction from the table above is that the great majority of the stars are extreme dwarfs apparently indicating that a star spends most of its life in the later stages of the spectral series."

It is really doubtful whether this important deduction can be made from these data alone. The relative frequencies in the sky today depend on three things: 1, the relative duration within each spectral class; 2, the distribution in space of the birthplaces of the stars and in time of the birthdays of the stars; and 3, the motions of the stars with respect to the solar system in the course of their evolution. The exact analysis of the effect of these three causes does not seem to have been given. If the chaos of the motions makes the third factor ineffective and if we can assume a constant "birth-rate" among the stars, then (and only then, I think) is the deduction of Shapley and Miss Cannon allowable.

The relative duration of the stars within the various spectral types must be related in some way to the relative frequency of stars of each spectral

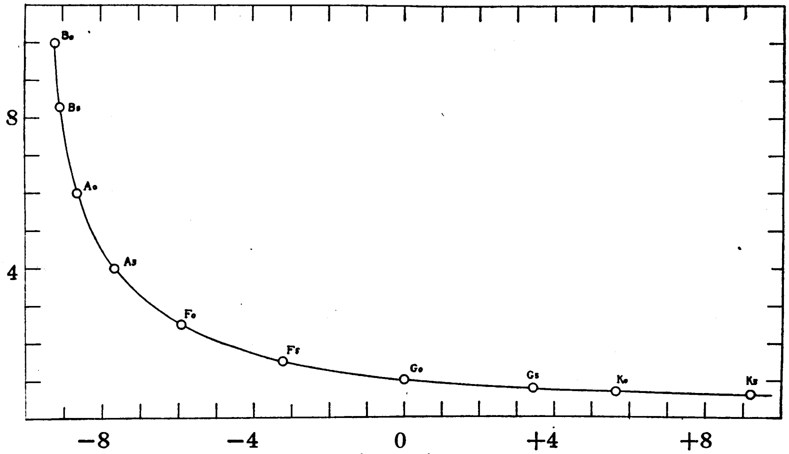

FIGURE 1

Mass of dwarf stars as a function of the time, the unit of the time (abscissæ) is 10^{12} years; that of the mass (ordinates) is the Sun's mass.

type as found in the sky today. If we suppose that new stars are being born at a steady rate and old stars are dying at a steady rate so that the population of the stellar universe is approximately constant, then we should expect to find the actual number of stars of each spectral class in a given volume at any time to be exactly proportional to the relative duration of an individual within each class.

Comparison with observation, however, will neither support nor discredit any particular calculation of relative duration in each spectral class. From exact knowledge of relative duration and from exact observation of the relative frequency in the sky today, we can only infer the distribution in time of the stellar birthdays.

While, therefore, no great weight is to be attached to the comparison, it is of interest to note that the frequency of the dwarfs of various types in a million cubic parsecs around the solar system, corresponds roughly to the relative durations as computed here.

[1] Shapley, Harlow, *On Radiation and the Age of the Stars. Pub. Astron. Soc. Pac.*, **31**, 178 (1919).

[2] Russell, H. N., *On the Sources of Stellar Energy. Ibid.*, **31**, 205 (1919).

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[5] Eddington, A. S. *On the Radiative Equilibrium of the Stars. Zeit. Physik*, **7**, 351 (1921).

[6] Seares, F. H. *The Masses and Densities of the Stars. Astrophys. J.*, **55**, 165 (1922).

[7] Shapley, H., and Cannon, A. J. *Summary of a Study of Stellar Distribution. Proc. Amer. Acad. Arts Sci.*, **59**, no. 9 (1924).

NOTES ON STELLAR STATISTICS. III: ON THE CALCULATION OF A MEAN ABSOLUTE MAGNITUDE FROM APPARENT MAGNITUDES, ANGULAR PROPER MOTIONS AND LINEAR RADIAL VELOCITIES

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A problem of frequent occurrence in stellar statistics is the calculation of the mean absolute magnitude of a group of stars of widely different apparent magnitudes when only the angular proper motions and the linear radial velocities are known. In what follows we shall assume that a solution for solar motion has been made and that the proper motions have been resolved into the υ components parallel to the sun's motion, and the τ components at right angles thereto and that the radial velocities have been corrected for the influence of this solar motion.

A procedure generally followed is the reduction of all proper motions to one and the same apparent magnitude, m_o. The arithmetic mean value of the reduced τ components is then compared with that of the radial velocities. Likewise, the algebraic mean of the υ components is compared with the total speed of the sun. Both comparisons will yield a value for the mean parallax of these stars, but, owing to the dispersion in absolute magnitude and in linear velocity the method is not entirely flawless. However, when sufficient material is at hand, it is possible to calculate a rigorous statistical correction to the results obtained in that way.

To facilitate computation we shall assume a Maxwellian distribution of